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QCD SPECTRUM FROM THE LATTICE

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ABSTRACT

Considerable progress has been made in the last year to derive the spectrum from QCD in the quenched approximation. I review the results for the proton to rho mass ratio and show that we are close to getting results to within 10%. I present new results for 0^{++} and 2^{++} glueballs. These results are qualitative since we do not address the question of mixing of glueballs with quark states. Finally, I give a status report on the full QCD calculations (2 flavors of dynamical Wilson fermions) being done on the Connection Machine 2.

INTRODUCTION

I would like to begin this talk with a semi-apology: Present Lattice QCD calculations do not predict numbers for the hadron spectrum (mesons, baryons, glueballs) that are reliable to better than 100 MeV. The reason for this is not that we have been idle (or more appropriately that the computers have been idle) but because so far we have been simulating an approximate theory – $SU(3)$ color without dynamical fermions ($n_f = 0$ flavor approximation). It may very well turn out that this approximation is very good for certain observables; unfortunately this justification can only come a-posteriori after we have simulated the real theory. On the other hand, to predict the glueball spectrum we definitely need to understand mixing with meson states. In this latter case quenched results should only be considered qualitative.

Quenched simulations provide an important reference point. Most of the software and numerical measurement techniques carry over unchanged to the real theory. Also, since pure gauge theory, $SU(3)$, is confining and asymptotically free it contains the qualitative essence of the real world. Further, chiral symmetry can be studied in the quenched approximation: the chiral behavior of observables can be derived and checked. It is therefore very important to get statistically significant numbers for the $n_f = 0$ world so that one can thereafter systematically examine the effect of quark loops in $n_f = 2, 3, 4$ simulations.

The notation used below is as follows: The gauge coupling is defined by $\beta = 6/g^2$. The quark mass m_q is given by κ for Wilson fermions, and m_s is the strange quark mass. I use a superscript v (d) for valence (dynamical) quarks when ever necessary. The spatial volume is denoted by N_s^3 and the temporal size by N_t such that the lattice size is $N_s^3 \times N_t$. Before

reviewing the results, let me first outline a list of technical points that we have to address in a systematic approach to getting reliable numbers from lattice QCD. This listing is also meant to provide you with the necessary criteria by which to judge lattice calculations.

1) Statistics: To get a good estimate of the path-integral, the data sample should cover all important regions of phase space. One way to overcome the possible presence of many disjoint but important regions of phase space is to use different starting points to generate separate Markov chains. The auto-correlation length for each such trajectory needs to be estimated and included in the error analysis. We can reliably estimate auto-correlation times for pure gauge update, but not so for update with quark loops. So at present, considerable effort is going into developing methods that reduce critical slowing down.

2) Finite Spatial Volume: A systematic study of the hadron and the glueball spectrum versus N_s has shown that the improvement in the signal with N_s compensates to a large extent the increase in the CPU time necessary to simulate the larger volume i.e. the errors are roughly constant for runs of constant CPU time. This is good news because in the calculation of the hadron spectrum a large N_s is necessary to take the chiral limit (reduce the extrapolation in m_q) while in glueball calculations a large N_s is necessary to remove mixing with toron states.

3) Finite N_t : Masses are extracted from the rate of decay of the 2-point correlation function examined at large separation in order to isolate the lowest mass state. One usually defines an effective mass as $M(t) = \log \left(\frac{\Gamma(t)}{\Gamma(t+1)} \right)$, where $\Gamma(t)$ is the 2-point correlation function. To make sure that the lowest state dominates the correlation function, $M(t)$ should approach a constant for t greater than some t_{min} . To see this asymptotic constant behavior the temporal lattice size has to be large and $N_t \geq N_s$. Exactly how large depends on the details of the calculations: for example (a) improved operators allow us to extract the asymptotic mass at smaller separation because they have a better overlap with the wavefunction; (b) the signal at large separation has errors which grow as $m_q \rightarrow 0$, consequently a very large N_t does not improve the mass estimates in the interesting limit. To get a reliable estimate, it is necessary that the effective mass $M(t)$ remains constant over ≥ 5 time slices and has small errors.

4) Finite lattice spacing a : The measurement of any physical observable on a coarse lattice has scaling violations that vanish only in the continuum limit. Since, in present calculations the lattice spacing is $a \approx 0.1$ fermi, therefore to show that we have control over lattice artifacts, mass ratios should remain constant as a function of β for a scale change of at least 2. Tests of scaling show that this translates to showing constant mass-ratios over the interval $6.0 < \beta < 6.4$ for the pure gauge theory [1]. The equivalent interval for $n_f = 2$ theory is likely to be $5.5 < \beta < 6.0$ for light quark masses i.e. $m_q^d < m_\pi$.

5) Improved Operators: If the interpolating operator used to extract the mass of a state matches the wavefunction of that state, then the exponential fall-off of the 2-point correlation function is dominated by a single exponential. In that case $M(t)$ is a good approximation to the answer even for small t . In the last few years we have made considerable progress in defining better operators and this has cleaned up the signal. So, estimates of the central value and of the errors on it have become more reliable.

6) Wilson versus Staggered Calculations: The lattice discretization of both the gauge part of the action and the Dirac operator is not unique. The simplest local version of the gauge action (Wilson's) has corrections of $O(a^2)$, while both Wilson and Staggered versions of the

Dirac operator have corrections of $O(a)$ but with different coefficients. Furthermore, Wilson and Staggered fermions have different properties with respect to chiral symmetry and flavor doubling. Wilson fermions do not have any doubling, and one assigns a 4 component Dirac fermion to each site on the lattice. Thus states with definite flavor can be constructed from local operators just like in the continuum. Unfortunately, the operator which removes the lattice degeneracy also breaks chiral symmetry explicitly. This does not effect mass calculations, but has proven to be a nuisance in the calculation of matrix elements between hadronic states.

Staggered fermions preserve a continuous $U(1)$ chiral symmetry but at the cost of 4-fold flavor doubling. Sixteen degrees of freedom (4 spin degrees times the 4 flavors) are represented as single component Grassmann variables on the sites of a 2^4 hypercube which forms the basic cell. Due to this spreading out, the spin and flavor quantum numbers are mixed up on the lattice at finite a . So the construction of operators is tedious and usually requires non-local operators. Also, the staggered flavor symmetry is broken at finite a . With an exact 4 flavor symmetry, there would be 16 Goldstone pions. Currently we find that even at $\beta = 6.0$, 15 of the pions are considerably heavier than the one Goldstone mode.

We expect that the effects of chiral symmetry breaking (Wilson) or flavor symmetry violation (staggered) to become small as $a \rightarrow 0$. Unfortunately, a quantitative evaluation of the dynamic restoration of these symmetry's requires detailed calculations. Our present guess is that these symmetries are restored to $\sim 10\%$ for $a < 0.1$ fermi. Because of the large differences between Wilson and Staggered fermion formulations, a check on lattice calculations is to demand consistency between the two results. The way this is usually expressed is to say that the deviations are $< X\%$ for $\beta >$ such and such. In this talk, I will show that we have made considerable progress in achieving this consistency.

7) Improved actions: The lattice actions can be modified by adding any number of irrelevant operators *i.e.* operators of dimension ≥ 5 which vanish as $a \rightarrow 0$. When the effect of these extra terms in the action is to improve the scaling behavior of observables, then such actions are called "improved". Unfortunately, so far we have not achieved much success in getting improved scaling by adding terms to either the gauge or fermion action. I feel that more work needs to be done, however, to systematically follow through an improvement program. This possibility will be explored in the coming years.

8) Algorithms for simulating dynamical fermions: Over the last two years considerable progress has been made in the development and understanding of algorithms to simulate QCD with dynamical quarks. The present algorithm of choice is the Hybrid Monte Carlo Algorithm (HMCA). It has certain drawbacks: with it we can only simulate a theory which has multiples of 2 degenerate flavors of Wilson fermions or multiples of 4 flavors of Staggered fermions. With this fermion algorithm the update is a factor of 100 – 1000 times slower than pure gauge, nevertheless, simulations on lattices of size $\sim 16^4$ have begun. The factor of 10 uncertainty comes from our poor understanding of long time auto-correlations in the updated lattices. In addition to HMCA, there does exist some data with the hybrid algorithm and the 2nd order Langevin. These algorithms are equally slow but allow for update with arbitrary number of flavors at the price of finite step size errors. The effects of these errors at small quark masses have not yet been investigated. Because of these drawbacks the present results with dynamical fermions should be considered preliminary.

The long term approach of lattice calculations to derive the hadron spectrum from QCD is to (a) get very accurate quenched results, (b) systematically investigate the effect of quark loops as a function of the quark mass and the number of flavors, and (c) to do realistic calculations at weak coupling and at small quark mass on large lattices. So let me first summarize the status of quenched calculations for the mesons, baryons and glueballs. Then I will present the status of our calculations with 2 flavors of Wilson fermions. Note that these calculations are still preliminary because the masses of dynamical quarks used in the update are still fairly heavy.

Quenched Spectrum:

Calculations of the spectrum in the quenched approximation began about 8 years ago. The touchstone for measuring progress has been the ratio R of the proton mass to the rho mass. This has in the past (until 1988) came out consistently high, usually > 1.6 . The measurements were, however, carried out at heavy quark mass, ($m_q \geq m_s$), and many of the criteria discussed above were not met. The situation has changed considerably in the last year due to improved measurement techniques and significantly more computer time. So, we are fast approaching the stage of providing definitive results in the quenched approximation.

In the real world we know two data points; (a) $R = 1.5$ for infinitely heavy quarks and (b) $R = 1.22$ for physical quarks. In between, where all lattice results lie, we can partly bridge the gap using phenomenological models. For heavy quarks, we can use potential models while for light quarks one should use the chiral quark model. Fitting these models to experimental data we can deduce the expected behavior as a function of quark mass. This is shown in fig. 1a and 1b as dark lines. I analyze the collective data from large lattice simulations in the quenched approximation against this background. This is shown in fig. 1a for Wilson fermions and fig. 1b for staggered fermions. These figures are called the APE invariant mass plot in which all dependence on the lattice spacing cancels out because only ratios of masses are used.

This new data show that results using Wilson fermions and from staggered fermions start to come together for $\beta \geq 6.0$. This is one of the consistency checks we had required.

The figures show a very significant trend; the ratio R decreases with increasing β . Already, at $\beta = 6.0$ the data fall on or even slightly below the phenomenological curves. If this trend continues as β is increased, then the quenched theory number will fall below the experimental value. This possibility should raise the eyebrows of the advocates of a large strange quark contribution to the mass of the proton. It has been conjectured that up to $400MeV$ of the proton mass comes from the strange sea. This is based on the mismatch between the experimental value of the pion-nucleon σ term ($50 - 60MeV$) and first order SU(3) breaking analysis ($\approx 26MeV$) [6] [7]. Clearly, what we measure in the quenched approximation are the masses with no sea quark contribution to any state. So the quenched ratio can lie on either side of the real world, depending on how large the sea quark contribution is to the proton versus the rho!

A second important feature of the data is the lattice size dependence. This can be seen in fig. 1a for Iwasaki *et.al.*'s data at $\beta = 5.7$ and for APE data at $\beta = 6.0$. Ironically, R decreases as the lattice size is increased for APE data but increases in Iwasaki's! Thus, unless we use a large lattice to extract the results, a meaningful behavior of M versus β cannot be deduced. This is perhaps the biggest reason why the spectrum data until recently has been so murky.

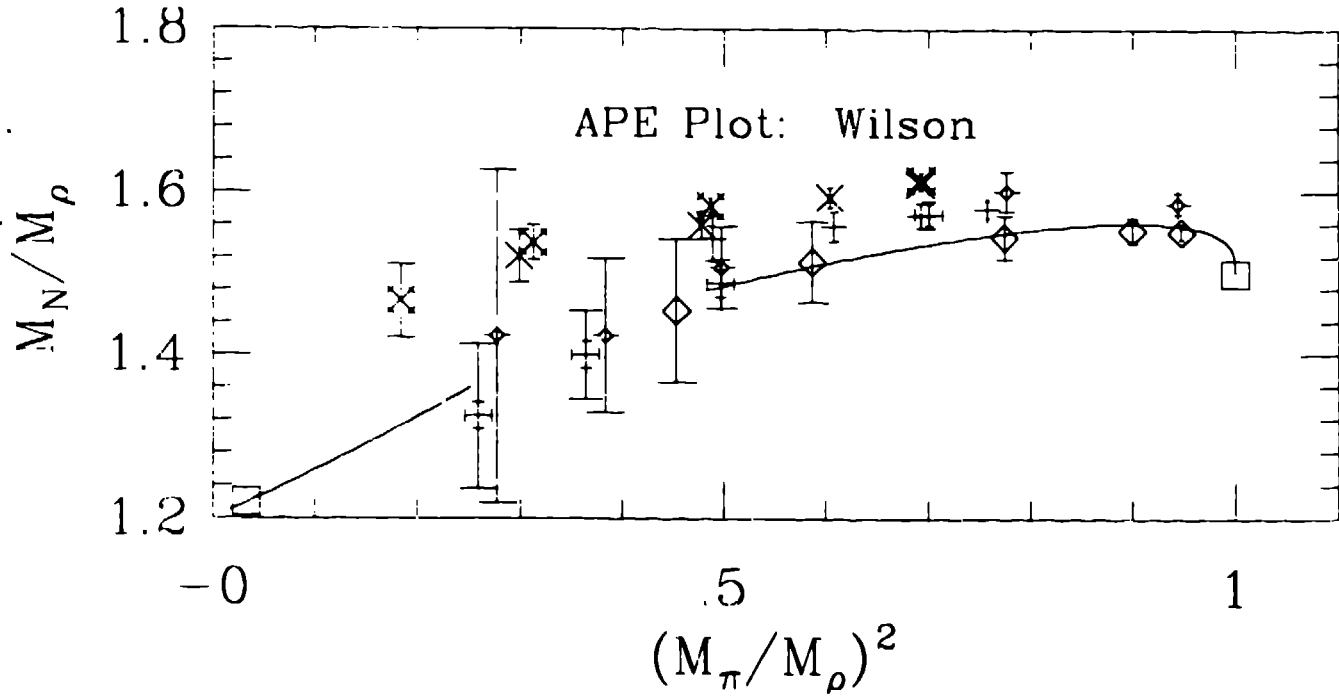


Fig. 1a: The APE mass plot for Wilson fermion data. Data at $\beta = 5.7$ is from the APE collaboration on $12^3 \times 24$ lattices (\times) and $24^3 \times 32$ lattices (fancy \times) [2]. The data at $\beta = 5.85$ is from Iwasaki et.al. on $16^3 \times 48$ lattices (\diamond) and $24^3 \times 60$ lattices (fancy \diamond) [3]. The data at $\beta = 6.0$ is also from the APE collaboration on $18^3 \times 32$ lattices (+) and $24^3 \times 32$ lattices (fancy +) [2].

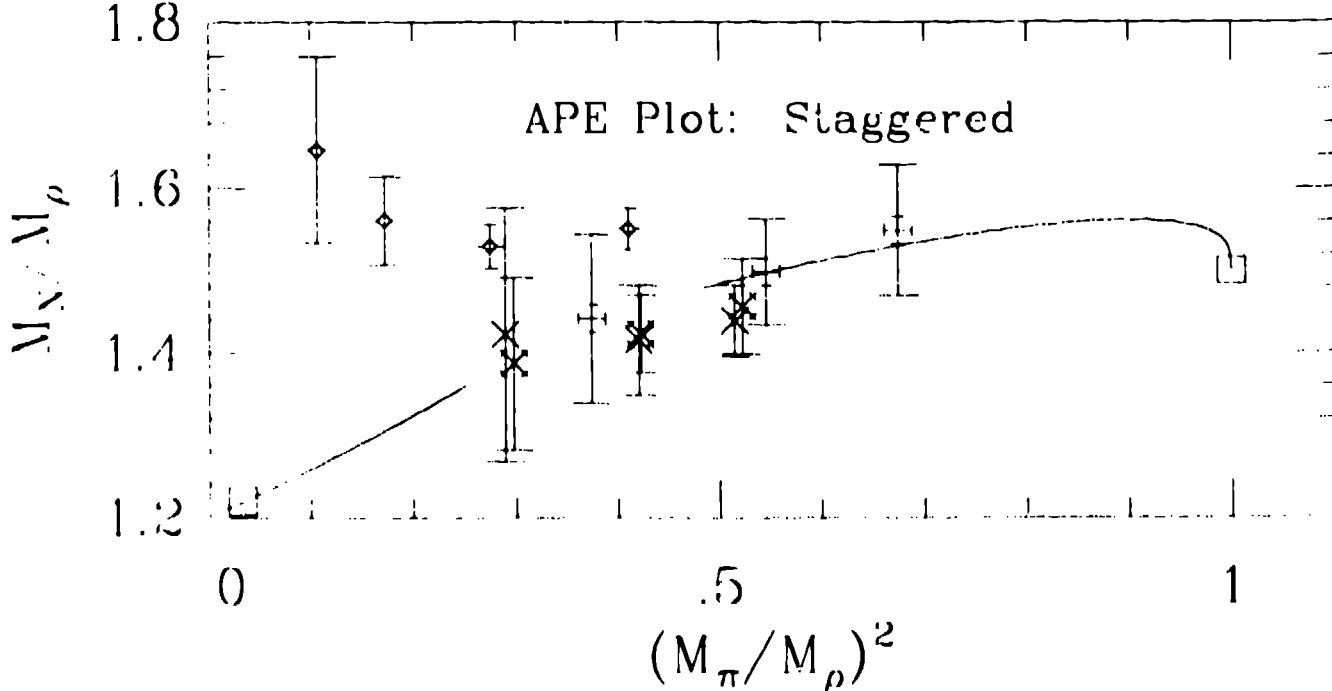


Fig. 1b: The APE mass plot for Staggered fermion data. Data at $\beta = 5.7$ is from the APE collaboration on $24^3 \times 32$ lattices (\diamond) [4]. The rest of the data is from the Staggered Collaboration; $\beta = 6.0$ on $16^3 \times 40$ lattices (\times) and $24^3 \times 40$ lattices (fancy \times), and $\beta = 6.2$ data is on $18^3 \times 42$ lattices (+) [5].

A measure of how well the lattice can reproduce hyperfine interactions is the splitting between the Δ and proton as a function of the quark mass. The experimental number for the ratio of the mass of the Δ to the proton is 1.31. The Wilson fermion data show that this ratio

is ≈ 1 for heavy quarks and increases as the quark mass is decreased (The signal for the Δ with staggered fermions is too poor to extract any numbers.). The ratio increases to ≈ 1.2 at the smallest quark mass in the APE data at both $\beta = 5.7$ and 6.0 . While this trend is encouraging, our enthusiasm has to be tempered by the fact that we do not know what the quenched result should be.

0^{++} and 2^{++} Glueball masses:

The best measured glueball states are the 0^{++} and 2^{++} . In units of the string tension, the estimates are [8] [9]

$$\frac{M_{0^{++}}}{\sqrt{\sigma}} \approx 3.4 \quad \frac{M_{2^{++}}}{\sqrt{\sigma}} \approx 5.3 \quad . \quad (1)$$

The errors on these numbers are quoted to be about 10%. Our present study has three goals: (a) to evaluate “improved actions” for better scaling; (b) to design better operators; and (c) to provide some information about the wave-function of these states.

The operators we use are “thick” Wilson loops and lines. These are constructed as follows: consider a simple Wilson loop, say 4×4 , as a template. Now replace each link by an average of it and its 4 spatial staples. This smears the loop and the averaging process can then be repeated using these thick links. The number of times each link is replaced by an average is called the smearing level. With each smearing the loop gets thicker. We expect the best results when we choose the basic template size to be the mean radius of the glueball. Then, a variation in the signal with the number of smearing levels will provide information about the distribution of color flux.

In figure 2, I show the data for $M(t)$ as a function of the loop size. The data show that the asymptotic mass is approached from above as it should. The significant noteworthy feature in the data is that the convergence of $M(t)$ versus t improves both with the loop size and the smearing level. This implies that the glueballs are large spatially extended objects. Our present best estimate for the masses is got from the largest loops we measured (4×4), and after each link has been smeared 4 times:

$$a\sqrt{\sigma}(5) = 0.222(15) \quad (2a)$$

$$aM_0^{++}(4) = 0.82(6) \quad (2b)$$

$$aM_2^{++}(3) = 1.21(9) \quad (2c)$$

These numbers give mass-ratios that are consistent with previous estimates. They have, however, been extracted from larger t than before. Converting to physical units using $\sqrt{\sigma} = 440 \text{ MeV}$, we get $M_0^{++} \sim 1.6 \text{ GeV}$ and $M_2^{++} \sim 2.4 \text{ GeV}$. The above numbers have unknown systematic errors which may be very large because (a) we have not included mixing with quark states, (b) the lattice size used is small, $12^3 \times 24$, and (c) the lattice is very coarse (all masses are ≈ 1 in lattice units). Therefore, I consider these estimates to be qualitative.

We are extending this calculation by measuring larger loops and more smearing levels. The calculation will also be extended to larger lattices and weaker couplings. The aim is to find the best operators in the quenched approximation before extending the calculation to full QCD.

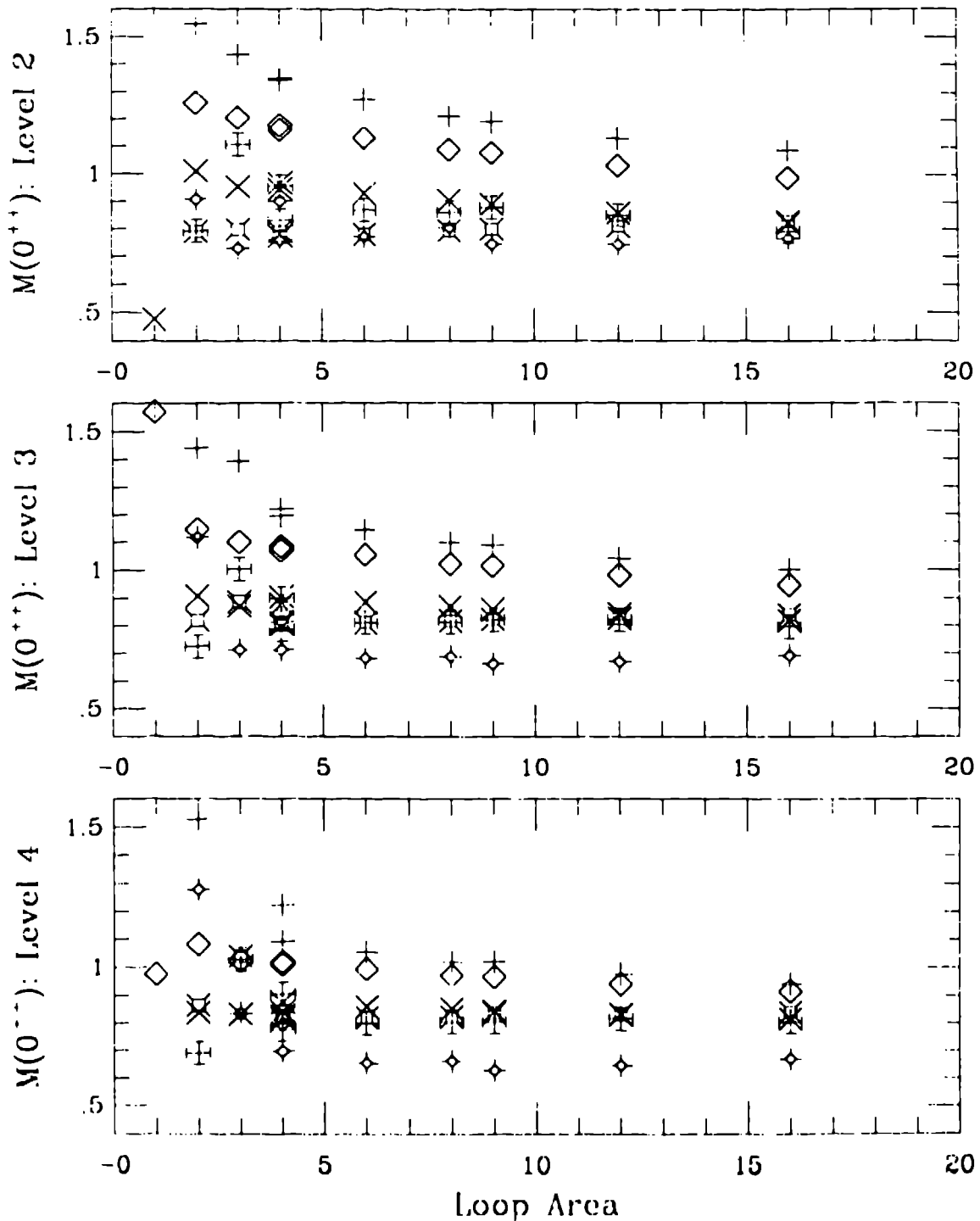


Fig. 2: *The effective mass for 0^{++} glueball versus loop area on smearing levels 2, 3, and 4. The data are for $t = 1$ (+), $t = 2$ (\diamond), $t = 3$ (\times), $t = 4$ (fancy square), $t = 5$ (fancy +), $t = 6$ (fancy \diamond) [10].*

Results for QCD with $n_f = 2$ flavors of Wilson fermions:

The LANL collaboration has undertaken a long-term systematic study to quantify the effects of quark loops. We will soon have data for $\beta = 5.3, 5.4, 5.5, 5.6$ for a variety of quark masses. The goal of this study is to calculate masses at as low a quark mass as possible with a given lattice size and then to look for trends as β is increased. This program is similar to the quenched case except for an additional complication. We do not a-priori know the value of the dynamical quark mass at which effects of quark loops will show up above statistical and

systematic errors. Looking at Wilson loop data (screening in the $q\bar{q}$ potential) a crude guess is that the effects will be manifest only for $m_q^d \leq m_s$. Since we are barely able to simulate at $m_q \approx m_s$ with present computer power, it may be 4-5 years before we can start quantifying the effects of dynamical fermions.

So, for the moment let me give you a feel for where we stand with respect to algorithm performance and a prognosis for what progress we can expect. In fig. 3 I show the world data for $\beta = 5.5$. The older calculation (\times) is by Fukugita *et.al.* who used a $9^3 \times 36$ lattice and a second order Langevin update algorithm [11]. The rest of the data are from the LANL group on a 16^4 lattice. This calculation is being done on the Connection Machine 2. These 16^4 lattices were produced using the Hybrid Monte Carlo Algorithm, which unlike the Langevin algorithm is an exact algorithm. If we linearly extrapolate the HMCA data taken at $\kappa = 0.158$ and 0.159 to $\kappa = 0.160$, we see a disagreement with the Langevin data. It is not easy to resolve whether the deviation is due to the approximate nature of the Langevin algorithm or due to finite volume effects. We will soon have data at $\kappa = 0.16$ with HMCA corresponding to $m_q \approx m_s$ and thereby make the deviation quantitative.

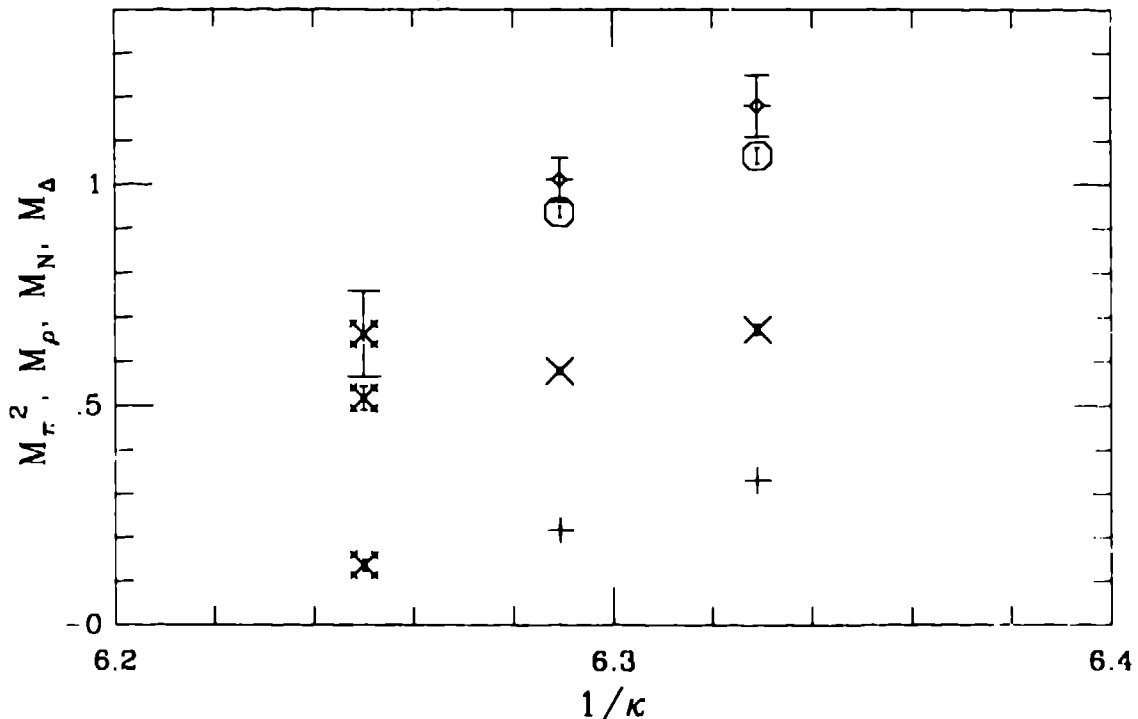


Fig. 3: Masses in lattice units versus $1/\kappa$ for $n_f = 2$ Wilson fermion simulations at $\beta = 5.5$.

The ratio R in our data is very similar to the quenched case for heavy quarks. In this regard we have not made much progress, however, the mere fact that we can generate configurations that incorporate the effect of dynamical fermions using an exact algorithm on lattices as large as 16^4 represents a significant step forward. In an earlier calculation done at stronger coupling ($\beta = 5.3$) [12] we did find evidence for large effects of sea quarks on masses. The present calculation will extend these results to weaker coupling.

Due to the fact that fermion update is slow, most calculations use an additional approximation. The lattices are doubled or tripled or quadrupled in time direction to obtain the asymptotic fall off the 2-point correlation function. For example, we doubled the 16^4 lattices

to $16^3 \times 32$ before calculating the quark propagators. Since we do not understand what systematic error this introduces, it behooves us to understand it in the quenched theory first. Let me conclude with an estimate of how much computer time is needed to simulate a world with $n_f = 2$ and $m_q = m_s$ on a $16^3 \times 32$ lattice. To generate 20 decorrelated lattices will require 1 Giga-flop year. This is clearly within our reach already.

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